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Comparing John von Neumann's assessment of Keynes's logical theory of probability with that of F P Ramsey

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Abstract

It is extremely important to recognize that von Neumann clearly and concisely recognized that Keynes' theory of logical probability was a nonstandard logic, while the two valued, "Boolean logic "of Jevons, Peirce and Schroder was a standard logic that was in conflict with the original logic and algebra of George Boole when it came to applications to probability. The Jevons-Peirce -Schroder(J-P-S) two valued "Boolean logic" REQUIRES linearity and additivity when applied to probability. This means that it can't deal with insufficiencies/deficiencies in the availability/completeness of the data or what Boole called indeterminate probabilities. Thus, the (J-P-S)" Boolean logic "is a standard logic, while Boole's original The Laws of Thought (1854) logic is a non-standard logic. Keynes's A Treatise on Probability was directly based on Boole's The Laws of Thought, and hence is a nonstandard logic. Thus, von Neumann is giving Keynes too much credit, as Keynes's nonstandard logic of 1921 is built directly on Boole's nonstandard logic of 1854. Keynes was also well aware of Boole's later, much easier, applications of indeterminate probabilities using Henry Wilbraham's approach (Keynes, 1921, p. 161).

Theodore Hailperin (1986, 1996) had already grasped what Boole's nonstandard approach was, which Boole modified shortly after The Laws of Thought was published in 1854 by switching to Henry Wilbraham's littoral's approach, which allowed one to bypass Boole's four valued logic, while obtaining the same results in a much easier manner.

Given Keynes's application of Boole's relational, propositional, logic, which was the foundation for his logical theory of probability, von Neumann's understanding of Keynes's work is complete and correct. It contrasts with Ramsey's understanding of Keynes's work which is nil.

Keywords: interval valued probability, logical probabilities, standard logics, non-standard logics, Ramsey's precise, additive, standard probability, von Neumann's non -precise, non-additive, non-standard probability

JEL Classification: D81

Section 1. Introduction

The paper will be organized in the following manner. Section Two will cover von Neumann's concise assessment of Keynes's logical theory of probability. Section Three will cover a few of Ramsey's many, many, many errors made in his work on Keynes between 1922 and 1926, as well as the very faulty foundation of Ramsey's work, which was to assume that Keynes's A Treatise on Probability (TP, 192) was based on a combination of G E Moore's Platonic Intuitionism as contained in his 1903 *Principia Ethica*, as well as Plato's metaphysical, speculative relations. All economists, etc., have followed Ramsey in his whimsical beliefs about Plato, Moore and Keynes.

How Ramsey' interpretation came to be universally accepted among economists, philosophers, historians, as well as by all other academicians working on Keynes's A treatise on Probability, is a mystery. This will require a comparison-contrast with von Neumann's assessment, unknown to economists, which was based on von Neumann's very deep understanding of (a) Keynes's graphical lattice illustration on page 39 of his book and (b) Keynes's worked out interval valued probability problems that calculated least upper bounds and greatest lower bounds, which are required in order to specify mathematical lattice structures, as contained on pp. 161-163 and 186-194 of chapters XV and XVII of the A Treatise on Probability(TP, 1921;1973, CWJMK version, p. 42), respectively

Section 2. von Neumann on Keynes's nonstandard, logical theory of probability in the A Treatise on Probability

Consider von Neumann's assessment of Keynes's work:

"To see how von Neumann's thinking on the foundations of probability changed, we turn next to an unfinished manuscript from about 1937, which is included in his Collected Works [44] … Von Neumann then makes the following declaration:

"We prefer, therefore, to disclaim any intention to interpret the relations P (a, b) =θ (0<θ<1) in terms of strict logics. In other words, we admit:

Probability logics cannot be reduced to strict logics, but constitute an essentially wider system than the latter, and statements of the form P(a, b)=θ (0<θ<1) are perfectly new and sui generis aspects of physical reality.

So, probability logic appears as an essential extension of strict logics. This view, the so-called 'logical theory of probability', is the foundation of J. N. [sic] Keynes's work on the subject. "

In short, the later von Neumann interprets quantum probabilities as logical probabilities. Moreover, he explicitly identifies this view with that worked out by Keynes. "(Stacey, 2018, pp. 3-4, underline added).

Von Neumann 's "strict logics" are standard logics that incorporate the assumption that all probabilities are linear and additive. Keynes's logical theory of probability is a non-standard logic that is" … an essential extension of strict logic." that is a "wider system "than the standard two valued logic of Jevons and Peirce.

Where, in Keynes's TP, did von Neumann come across materials written by Keynes that convinced von Neumann that Keynes had presented a generalization of the Classical, additive approach to probability?

There are two different places in TP where this takes place. First, we have Keynes's graphical illustration presented on page 39 of chapter III of the TP.

A careful reading of von Neumann's 1937 discussion of his decision to adapt Keynes's logical approach to probability, in order to discuss quantum probability, shows that von Neumann had carefully read Keynes's Part I, pp36-40 and Part II, pp. 160-163 and pp186-194. These are the pages that Keynes applied Boole's mathematical lattice structure dealing with least upper bounds and greatest lower bounds. von Neumann grasped that Keynes had correctly generalized the standard mathematical laws of the calculus of probability in order to incorporate interval valued probability, such as his non numerical probabilities. Consider the following examples. Let p_1= [.47, .55] and p_2= [.54, .60] An analysis of these two non-numerical probabilities follows directly from Keynes's analysis as stated in his TP:

> "…I maintain, then, in what follows, that there are some pairs of probabilities between the members of which no comparison of magnitude is possible; that we can say, nevertheless, of some pairs of relations of probability that the one is greater and the other less, although it is not possible to measure the difference between them; and that in a very special type of case, to be dealt with later, a meaning can be given to a numerical comparison of magnitude. I think that the results of observation, of which examples have been given earlier in this chapter, are consistent with this account. By saying that not all probabilities are measurable, I mean that it is not possible to say of every pair of conclusions, about which we have some knowledge, that the degree of our rational belief in one bears any numerical relation to the degree of our rational belief in the other; and ***by saying that not all probabilities are comparable in respect of more and less, I mean that it is not always possible to say that the degree of our rational belief in one conclusion is either equal to, greater than, or less than the degree of our belief in another***." (Keynes, 1921, p. 34; boldface, italics, and underline added).

Now by "numerical", Keynes means by a single number. Keynes will show that "non numerical" probabilities, intervals like p_1 and p_2, can provide support for rational belief

Note that there are an infinite number of examples as the one I gave above. Therefore, following Keynes, it is not possible to generally say that $p_1>p_2$, $p_1<p_2$ or $p_1=p_2$. p_1 and p_2 are both indeterminate probabilities that overlap one another.

Of course, Keynes, who mainly used the terminology approximation, inexact measurement and non-numerical probability, is talking about what are today called imprecise or non-additive probability, just as Boole's indeterminate probabilities are interval probabilities that are partially ordered. Keynes is defining, just as Boole did, a partial ordering of probabilities in the probability space by his constant use of the word "between " in chapter III of the A Treatise on Probability, which corresponds to an ordering based on the inequality, ≤, and leads to the illustration by Keynes of a mathematical lattice structure on p. 39 of chapter III in the TP. H. E. Kyburg demonstrated this FOUR times, in 1994, 1999, 2003, and 2010, without having any knowledge about Keynes's use of Boole's mathematical apparatus to specify interval valued probability, as discussed by Keynes in Part II in the appendix to chapter XIV, chapter XV, chapter XVI, and chapter XVII of the TP. Following Boole, Keynes is using propositions about events and not the events themselves to discuss probability. Second, Keynes is using Boole's relational, propositional logic that uses the logical connectives "and", "not" and "or" to analyze sets of conjunctions of propositions and sets of disjunctions of propositions. Third, Keynes is calculating least upper bounds (l. u. b's) and greatest lower bounds (g. l. b. 's) for the sets of conjunctions and disjunctions, just as Boole did on pp. 268-325 of *The Laws of Thought*, that, in modern terminology, specify a mathematical lattice structure in Euclidean space.

This approach is termed by von Neumann to be a "wider system" than a classical "strict system", which incorporates the standard additivity assumption. It is a non-standard probability approach because it incorporates non-additivity. This non-standard probability approach is illustrated by Keynes with the interval probability paths U, V, W, X, Y, and Z, where 0VA is an explicit interval probability, and the standard, strict probability path is given by 0AI. Keynes's illustration is of a mathematical lattice structure, which is comprised of the nonlinear, non-additive interval probabilities U, V, W, X, Y, and Z, plus the additive and linear probabilities given by 0 AI, where A is an additive and linear probability. Thus, OAI represents a limited, lattice structure, where all outcomes are defined on the set of real numbers between 0 and 1, that is consistent with the Jevonian reinterpretation of Boole's original approach. Boole's original approach combined his new relational, propositional logic with the basic laws of mathematical algebra. This allowed Boole to develop a mathematical lattice approach to estimate indeterminate(imprecise) probability as a generalization of determinate(precise) probability. Jevons's approach eliminated Boole's use of standard algebra techniques from consideration so that the lattice constructions developed by Boole to represent both precise and imprecise probability could now only be used to represent precise (additive and linear) probability. Starting in 1933, Garrett Birkhoff's work updated, reorganized and relabeled concepts, such as least upper bounds and greatest lower bounds that Boole used in his original approach, by using a new terminology to describe Boole's approach. For example, Birkhoff was the first to use terms such as "lattice, poset (partially ordered set), Join, Meet, Orthocomplemented lattice, etc. "Birkhoff thus reorganized this branch of mathematics so that Boole's imprecise intervals could again be specified.

Von Neumann's discussion shows that von Neumann understood this. However, von Neumann gave the credit for this new nonstandard approach to Keynes. He was then able to generalize from Keynes's Euclidean space to a Hilbert space, where Keynes's paths are replaced by subspaces in Hilbert space in order to model a propositional logic leading to a mathematical lattice structure based on lub's and glb's, some of which are classical Boolean structures that are additive and linear; however, overall, the lattice structure is nonstandard as it incorporates imprecise probability.

Again, it is important to note, as was first pointed out by Hailperin (1986, 1996), that the Boolean approach being used by Keynes is NOT the Jevons and Peirce" Boolean "interpretation based on a two valued (1, 0) logic. Boole's approach was based on a four valued logic that allowed Boole to deal with vagueness, ambiguity, uncertainty, and unavailable, incomplete and missing evidence. This is not possible in the Jevons-Peirce approach, but is possible in the Birkhoffian rewriting and updating of Boole's approach.

Finally, von Neumann has implicitly defined interval probabilities by use of the basic notation P (a, b), where P (a, b) is von Neumann's notation for Keynes's logical probability construct. In 1938, von Neumann (Stacy, 2016, p. 5) gives the following restated assessment of von Neumann's unpublished, 1937 statement:

"A complete derivation of quantum mechanics is only possible if the propositional calculus of logics is so extended, as to include probabilities, in harmony with the ideas of J. M. Keynes. In the quantum mechanical terminology: the notion of a « transition probability »from a to b, to be denoted by P(a, b) must be introduced. P(a, b) is the probability of b, if

a is known to be true. *P(a, b) can be used to define a≦b and −a:* P(a,b)=1 means a≦b,P(a, b) =0 means a≦−b. (But P(a, b)=φ, with a φ>0, <1 is a new « sui generis » statement, only understandable in terms of probabilities.) [46, p. 38; italics added]." (Stacey, 2016, p. 6)

Note that in the October, 1936 article by Birkhoff and von Neumann, it is stated that

"Again, such a lattice is a single projective geometry if and only if it is irreducible-that is, if and only if it contains no "neutral" elements…such that a = (a ∩ x) ∪ (a ∩ x') for all a. In actual quantum mechanics such an element would have a projection-operator, which commutes with all projection-operators of observables, and so with all operators of observables in general. This would violate the requirement of "irreducibility" in quantum mechanics. [28]Hence we conclude that the *propositional calculus of quantum mechanics has the same structure as an abstract projective geometry*." (Birkhoff and von Neumann, 1936, p. 833-note that an entire appendix is provided on this topic from pp. 837-843).

Thus, in the October, 1936 article, Birkhoff and von Neumann viewed quantum probabilities as measures over closed subspaces of Hilbert space. Using continuous (projective) geometry, he began to treat probability—specifically the transition probability—as a primitive geometric relation (comparable to "angle") between elements in a lattice, rather than a separate measure added onto the system. Thus, the probabilities would appear as numerical values (typically the modulus-square of the inner product) which can't be viewed as a classical probability, since classical, standard, mathematical probability requires additivity and linearity. von Neumann's later reflections in 1937 and 1938 suggest he then viewed P(a, b) as representing an imprecise (indeterminate) probability, following in the footsteps of George Boole and Keynes.

We can conclude that von Neumann, much like Edgeworth in 1922, gave an excellent short and concise summary of Keynes's logical theory of probability showing that it is a generalization of the strict logic of pure mathematical probability, which is based on additivity and linearity, and hence has a wider applicability. On Neumann's

P(a, b) =θ (0<θ<1) is equivalent to Keynes's P(a/h)= α, which is a mathematical version of Boole's argument form (See Keynes, 1921, p. 119).

Thus, in their 1936 paper, Birkhoff and John von Neumann change the interpretation of the Born rule by shifting the focus from wavefunction amplitudes to the algebraic structure of a relational, propositional logic itself. The Born rule originally describes probability as the square of a complex amplitude (Ψ^2). Birkhoff and von Neumann's paper modifies this into a more fundamental geometric and logical requirement by replacing the view that quantum mechanics is analyzed through wavefunctions and instead analyzes it as a system of propositions, sentences or statements made about experimental events or outcomes.

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These propositions are represented as being closed, linear subspaces of a Hilbert space.

Instead of asking for the probability of a particle being in a specific location state of a wavefunction, they ask for the probability that a specific "proposition" about the event (measurement outcome) is true. In other words, they move from an analysis of quantum outcomes or events to an analysis of logical propositions as being logical statements made about quantum outcomes or events.

Birkhoff and von Neumann showed that the "logic" of quantum mechanics is non-distributive, meaning it doesn't follow standard classical rules. In this framework, probabilities are measures assigned to propositions about the subspaces of Hilbert space.

Section 3. F P Ramsey on Keynes's logical theory of probability in the *A Treatise on Probability*

I will give two examples of Ramsey's very severe errors. All of these errors are due to Ramsey's complete failure to grasp Keynes's use of Boole's relational, propositional logic. A relational, propositional logic means that the propositions must be related internally. Ramsey, instead, interprets Keynes's system as being a propositional logic dealing with *unrelated* propositions. This makes no sense. In fact, it is nonsense. At no time in his life did Ramsey ever grasp that probabilities must be related to evidence in any logical theory. In such a relational construction, there MUST BE an objective, logical, probability relation connecting the h premises to the conclusions, internally. Thus, using Keynes's page 119 discussion in the TP, we have

$P(a/h) = \alpha, 0 \leq \alpha \leq 1$,

where P denotes the logical, objective, probability relation between the a and h propositions, where a is the conclusion, h are the premises considered true and α denotes the partial, rational degree of probable belief holding between h and a.

Consider the following statement from Ramsey in 1922 in *Cambridge Magazine:*

"First, he thinks that between any two non-self-contradictory propositions there holds a probability relation (Axiom I), for example between 'My carpet is blue' and 'Napoleon was a great general'; it is easily seen that it leads to contradictions to assign the probability 1/2 to such cases, and Mr. Keynes would conclude that the probability is not numerical. But in such cases there is no probability; that, for a logical relation, other than a truth function, to hold between two propositions, there must be some connection

between them. If this be so, there is no such probability as the probability that 'my carpet is blue' given only that 'Napoleon was a great general', and there is therefore no question of assigning a numerical value." (Ramsey, 1922, pp. 3-4).

First, there is no axiom I in Keynes's TP that states that

"…between any two non-self-contradictory propositions there holds a probability relation (Axiom I), for example between 'My carpet is blue' and 'Napoleon was a great general';" (Ramsey, 1922, pp. 3-4; note that this error is repeated in Ramsey's 1926 contribution, published in 1931)

Second, Ramsey's "example" consists of *two unrelated* propositions.

Third, Keynes's logic is not limited to two propositions and *does not apply to any pair or set of unrelated propositions.*

Let us take an example from Ramsey's 1926 paper, *Truth and Probability*:

"But let us now return to a more fundamental criticism of Mr. Keynes' views, which is the obvious one that there really do not seem to be any such things as the probability relations he describes. He supposes that, at any rate in certain cases, they can be perceived; but speaking for myself I feel confident that this is not true. I do not perceive them, and if I am to be persuaded that they exist it must be by argument; moreover, I shrewdly suspect that others do not perceive them either, because they are able to come to so very little agreement as to which of them relates any two given propositions." (Ramsey. 1926. In Kyburg and Smokler, (eds.)1980(2nd ed.), pp. 27-28-This bizarre rant was already obliterated by Boole on pp. 7-8 of LT. Keynes cites Boole on page 5 of the TP in a footnote).

ALL of Ramsey's examples of logical probability entail two unrelated propositions.

It is quite impossible for a relational, propositional logic to deal with Ramsey's pairs of unrelated propositions, which is why such unrelated propositions are excluded by definition by Boole and Keynes.

The great mystery here is how it was possible for all economists, philosophers, statisticians, historians, social scientists, behavioral scientists, etc., who had published on Keynes's work on probability, to have accepted Ramsey's obviously wrong arguments for over one hundred years.

It should come as no surprise to the reader that the latest work on Keynes's theory, by M. Coates (2025), F J Aristimuno (2025), and C. Zappia (2025), are confused works, which are guaranteed to further confuse their readers, when Keynes's contributions are being discussed because they accept Ramsey's claims that Keynes's approach deals with unrelated pairs of outcomes.

Ramsey's errors, however, are an example of a shrewd observation made by von Neumann and Morgenstern in 1944:

"There is no point in using exact methods where there is no clarity in the concepts and issues to which they are to be applied. Consequently, the initial task is to clarify *the knowledge of the matter by further careful descriptive work."* (von Neumann and Morgenstern, 1944; italics added).

Ramsey's work on Keynes was carried out with no knowledge on Ramsey's part about what Keynes was doing, which was providing an analysis of related sets of propositions. Ramsey needed to have spent time actually reading Keynes's TP before writing about it in ignorance. This is also very good advice to the economists, philosophers and historians writing about Keynes in 2025.

Section 4. Conclusions

Von Neumann made a major contribution, although von Neumann's contribution is completely unknown to economists, philosophers and historians, to the study of Keynes's logical theory of probability in the TP. Von Neumann realized that Keynes had successfully generalized classical probability theory. However, von Neumann overlooked the important role of George Boole, as Keynes's contribution was built on the foundations of Boole's 1854 work, The Laws of Thought.

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F P Ramsey's never made a single contribution to the study of Keynes's logical theory of probability in the TP. Ramsey 's work about Keynes is based on fictional and fictitious claims that were never supported by any recognition that Keynes's work in the TP was based on George Boole's LT. Instead of Boole, Ramsey substituted Plato and Moore. Ramsey came to the highly illogical conclusion that Keynes's TP was all about unrelated propositions, even though Keynes had made it crystal clear that the propositions must be related.

As H E Kyburg had pointed out four times, in 1994, 1999, 2003 and 2010, in work ignored by all economists, philosophers, and historians working on Keynes's TP approach, Ramsey never discussed Keynes's work on pp. 34-40 of the TP. The reason is obvious. Ramsey's theory of subjective probability is represented by the horizontal line, 0AI. This is simply the standard, classical, *"strict"*"logic requiring additivity. Ramsey's 0AI (see Keynes, 1921, p. 39), based theory can't deal with uncertainty, non-additivity, non-linearity or vagueness. Von Neumann realized that Ramsey's theory COULD NOT deal with quantum probability. Only Keynes's logical theory could deal with it because Keynes's theory alone connects Boole's original approach, based on what Birkhoff, who was the first mathematician to use the term "lattices", to imprecise probability.

In conclusion, Ramsey's lattice structure is based on the two valued logic of Jevons and Peirce that requires additivity and linearity. One can only use the real numbers between 0 and 1. One can't use intervals because they are not single, precise real numbers between 0 and 1. The line OAI in Keynes's illustration on page 39 of the TP is Ramsey's lattice.

Keynes's lattice structure, described by Kyburg as being 'a much richer manifold …', includes the precise, numerical probabilities OAI line of Ramsey plus the non-numerical probabilities U, V, W, X, Y, Z.

The work of Birkhoff (1933, 1936, 1940) allows one to reintegrate Boole's original indeterminate probabilities and Keynes's non numerical probabilities into lattice structures by reformulating lattice theory so as to integrate imprecise probabilities into lattice structures, which were removed by Jevons and Peirce when they attempted to reinterpret Boole's work, which they characterized as being mysterious.

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