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The connections between Keynes's 1921 logic of probability and the Birkhoff-von Neumann 1936 logic of quantum mechanics

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Abstract

Birkhoff and von Neumann make it clear what the goal of their October, 1936 paper, "The logic of quantum mechanics", is: "The object of the present paper is to discover what logical structure one may hope to find in physical theories which, like quantum mechanics, *do not conform to classical logic*. Our main conclusion, based on admittedly heuristic arguments, is that one can reasonably expect to *find a calculus of propositions* which is formally indistinguishable from the calculus of linear subspaces with respect to *set products, linear sums, and orthogonal complements*-and resembles *the usual calculus of propositions with respect to and, or, and not.*" (Birkhoff and von Neumann, 1936, p.823; underline added).

Specifically,

"Up to now, we have only discussed formal features of logical structure which seem to be common to classical dynamics and the quantum theory. We now turn *to the central difference between them-the distributive identity of the propositional calculus*:

L6: a U (b ∩ c) = (a U b) ∩ (a U c) and a ∩ (b U c) = (a ∩ b) U (a ∩ c)

which is a law in classical, *but not in quantum mechanics*." (Birkhoff and von Neumann, 1936, p.830; underline added)

Continuing, Birkhoff and von Neumann comment:

"From a deeper mathematical viewpoint, L6 is the characteristic property of set-combination. More precisely, every "field" of sets is isomorphic with a Boolean algebra, and conversely.[21] This throws new light on the well-known *fact that the propositional calculi of classical mechanics are Boolean algebras." (*Birkhoff and von Neumann, 1936, p.831).

Birkhoff and von Neumann then show on p.831 that the distributive law breaks down in quantum mechanics and that the generalized distributive law of logic breaks down in the related field "in the quotient algebra of the field of Lebesgue measurable sets by the ideal of sets of Lebesgue measure 0, which is so fundamental in statistics and the formulation of the ergodic principle." (ibid., p.831).

However, it must be noted that Birkhoff and von Neumann are talking about the Jevons-Pierce-Schroder interpretation (two -valued) of Boole's original approach (four -valued), which Boole completely rejected. (See Hailperin, 1986, 1996), The reason for this rejection is that the Jevons-Peircean approach required strict complementation, which translated into additivity and linearity in probability assessments, as opposed to Boole's realization that real world decision making required sub additivity and non-linearity, Thereafter, we will refer to the Jevons-Pierce-Schroder approach as Jevons's approach to the interpretation of Boolean logic, which uses standard classical logic to underpin discussions of standard probability logic. Jevons's approach removed the basic algebra which Boole combined with his new relational, propositional logic, so as to deal with indeterminate, imprecise probabilities, which are nonstandard. Jevons's approach can't deal with the non-standard probability logics used by Boole in chapters XVIII-XX of *The Laws of Thought* (1854) and by Keynes in chapters XII, XV-XVII of the *A Treatise on Probability* (1921) in order to deal with indeterminate, interval valued probabilities.

Keywords: relational propositional logic, standard probability logics (additive and linear, Jevons, Peirce, Schroder, Boolean, two valued logics), non-standard probability logics (non-additive, nonlinear, Boole-Keynes four valued logics), imprecise probability, interval probability, upper-lower probability, indeterminate probability non-numerical probability, mathematical lattice structures, inapplicability of distributive law

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Section 1. Introduction

The paper will be organized in the following manner, Keynes's approach to logical structure will be presented first in Section Two. It is based on the concept of a proposition about an event, outcome or occurrence, as opposed to the physical event or outcome itself, sets of such propositions, a partial ordering of the propositions based on ≤ or betweenness, and a relational, propositional logic incorporating logical implication that uses logical operators (and, or, not) for dealing with the sets of the conjunctions and disjunctions of the propositions. This results in the specification of least upper bounds and greatest lower bonds, resulting in an algebraic non -linear, non-additive mathematical lattice structure defined on a Euclidean n- space, that Keynes presented diagrammatically on page 39 of his A Treatise on Probability. This will then lead to Keynes's general rejection of the generality of the application of precise, exact mathematical expectations calculations in economics, with such calculations being restricted to applications based on the type of case represented by the linear, additive line, OAI, in the diagram on p.39.The linear line, OAI, represented F P Ramsey's subjective expected utility theory, which can be seen to be a very special case of Keynes's general theory of decision making under conditions of both uncertainty(partial knowledge-partial ignorance) and risk(complete knowledge).In chapter XXVI of the A Treatise on Probability(TP, 1921) and General Theory in chapters 11 and 12, Keynes rejected the standard calculation rules for mathematical expectation, which rely fundamentally on the distributive law (along with linearity of sums and scalar multiplication). In fact, the linearity of expectation is essentially a probabilistic restatement of distributivity. The expectation operator is defined using sums (or integrals). Because sums and integrals obey the distributive law, expectation inherits the same structure. In its place, he put inexact, interval valued probabilities and decision weights like his conventional coefficient of weight and risk. von Neumann also rejected the generality of the distributive law due to non-linearity and non-additivity. This is one of the reasons we will find for von Neumann's statements about his building on Keynes's approach.

Section Three will cover the much more detailed, mathematically and logically complex, system of Birkhoff-von Neumann, where the propositions, like Keynes's, are based on the observations (observables) of physical systems. Birkhoff and von Neumann represented their model by the use of a mathematical phase space. The propositions about the physical observables are then defined as subsets of the phase space. This leads to a direct break from classical modeling, where

"…before a phase-space can become imbued with reality, its elements and subsets must be correlated in some way with "experimental propositions" (which are subsets of different observation-spaces). Moreover, this must be done so that set-theoretical inclusion (which is the analogue of logical implication) is preserved. There is an obvious way to do this in dynamical systems of the classical type. 'One can measure position and its first time-derivative velocity-and hence momentum-explicitly and so establish a one-one correspondence which preserves inclusion between subsets of phase-space and subsets of a suitable observation-space. In the cases of the kinetic theory of gases and of electromagnetic waves *no such simple procedure is possible*..." (Birkhoff and von Neumann, 1936, p.825; italics added)

Therefore,

"…a thoughtful analysis shows that another and more subtle idea is involved. The central idea is that physical quantities are *related, but are not all computable from a number of independent basic quantities (such as position and velocity).'*

We shall show …that this situation has an exact algebraic analogue in the calculus of propositions." (Birkhoff and von Neumann, 1936, p.825; underline added). Birkhoff and von Neumann then follow in the footsteps of Keynes's A Treatise on Probability (TP, 1921) on

- Partial orders or partially ordered systems
- Specification of least upper bounds (lub's)and greatest lower bounds(glb's) for the intersections and unions of sets of propositions
- Mathematical lattice structures
- Complemented lattices, which are the algebraic set theoretic analogue for the logical operator, "not", used in the relational, propositional logic
- Breakdown of the distributive law
- Relation to abstract, projective geometry, compared to Keynes's abstract, Euclidean geometry. Birkhoff and von Neumann conclude that "Hence we conclude that the propositional calculus of quantum mechanics has the same structure as an abstract projective geometry.
  Moreover, this conclusion has been obtained purely by analyzing internal properties of the calculus, in a way which involves Hilbert space only indirectly." (Birkhoff and von Neumann, 1936, p.833).

Section Four will deal with how Keynes's mathematical skill set has been assessed by economists. We will find that there is a major conflict between the assessments made by von Neumann of Keynes and the assessments of the economists. Section Five concludes the paper.

Section 2. Keynes's logical structure

The major error made in academia for over at least 105 years, especially by philosophers, economists and historians, is to assume that Keynes was the founder of the logical theory of probability. The first logical theory of probability was that of Thomas Aquinas. The second, much more mathematically and technically advanced logical theory of probability, was created by George Boole. Keynes's logical theory is built on Boole's theory,

Keynes first makes it clear that he is dealing with sets of propositions about events or outcomes and not the events or outcomes themselves on page 5. In a footnote on p.5, he makes it clear that the use of propositions in an argument form was emphasized by Boole. Secondly, Keynes makes it clear that he is dealing only with partial ordering of probability space and not a complete order as used by economists:

> "I maintain, then, in what follows, that there are some pairs of probabilities between the members of which no comparison of magnitude is possible; that we can say, nevertheless, of some pairs of relations of probability that the one is greater and the other less, although it is not possible to measure the difference between them; and that in a very special type of case, to be dealt with later, a meaning can be given to a numerical comparison of magnitude. I think that the results of observation, of which examples have been given earlier in this chapter, are consistent with this account. By saying that not all probabilities are measurable, I mean that it is not possible to say of every pair of conclusions, about which we have some knowledge, that the degree of our rational belief in one bears any numerical relation to the degree of our rational belief in the other; and by saying that not all probabilities are comparable in respect of more and less, I mean that it is not always possible to say that the degree of our rational belief in one conclusion is either equal to, greater than, or less than the degree of our belief in another." (Keynes, 1921, p.34).

Note that the examples mentioned by Keynes above, that were covered by Keynes on pp.22-34 of the A Treatise on Probability, "I think that the results of observation, of which examples have been given earlier in this chapter, are consistent with this account.", are all interval valued examples. Keynes continues the theoretical discussion started late in chapter III in chapter XII with his axiom (i), which involves a more detailed discussion of Boole's relational, propositional logic:

"We shall assume that there is included in every premiss with which we are concerned the formal implications which allow us to assert the following axioms:

(i.) Provided that a and h are propositions or conjunctions of propositions or disjunctions of propositions, and that h is not an inconsistent conjunction, there exists one and only one relation of probability P between a as conclusion and h as premiss. Thus any conclusion a bears to any consistent premiss h one and only one relation of probability." (Keynes, 1921, pp.134-135; underline added).

Given sets of conjunctions (based on the logical operator "and") and disjunctions of propositions (based on the logical operator "or"), what are now called, following Birkhoff, the Meet(intersections) and Join(unions), can be calculated, The glb is the Meet and the lub is the Join.

The stage is now set for Keynes to apply his version of the original Boolean algebra, which followed from Henry Wilbraham's improvements in technical exposition in 1854 that Boole integrates into his work starting in 1854- 1855.See the footnote on page 160 of the TP for Keynes's discussion of Boole's 1854 approach in LT. This means that Keynes has recognized that the algebraic analogue for the use of the logical operator "not", in Boole's relational, propositional logic, is what is now called "complementation ", or the set complement, when applying Boole's revised algebraic approach based on Wilbraham.

Keynes's derivation of lub's and glb's appear in chapters XV, pp.162-163 and chapter XVII, pp.186-189 and pp.190-19. These mathematical derivations by Keynes can then be compared to Kyburg's graphical derivations in 1995, 1999, 2003 and 2010. Both types of derivations lead to the same conclusion-Keynes has systematically presented a mathematical lattice structure to represent his non numerical system of (interval valued) logical probabilities, which was recognize by von Neumann in 1937.

The result of Keynes's analysis is discussed in chapter XXVI of the TP:

> "In Chapter III. of Part I. I have argued that only in a strictly limited class of cases are degrees of probability numerically measurable. It follows from this that the 'mathematical expectations of goods or advantages are not always numerically measurable; and hence, that even if a meaning can be given to the sum of a series of non-numerical 'mathematical expectations,' not every pair of such sums are numerically comparable in respect of more and less. Thus even if we know the degree of advantage which might be obtained from each of a series of alternative courses of actions and know also the probability in each case of obtaining the advantage in question, it is not always possible by a mere process of arithmetic to determine which of the alternatives ought to be chosen. If, therefore, the question of right action is under all circumstances a determinate problem, it must be in virtue of an intuitive judgment directed to the situation as a whole, and not in virtue of an arithmetical deduction derived from a series of separate judgments directed to the individual alternatives each treated in isolation. We must accept the conclusion that, if one good is greater than another, but the probability of attaining the first less than that of attaining the second, the question of which it is our duty to pursue may be indeterminate, unless we suppose it to be within our power to make direct quantitative judgments of probability and goodness jointly.

It may be remarked, further, that the difficulty exists, whether the numerical indeterminateness of probability is intrinsic or whether its numerical value is, as it is according to the Frequency Theory and most other theories, simply unknown." (Keynes, 1921, p.312).

This means that only the linear and additive line OAI of the illustration on p.39 of the TP represents probabilities where strict or exact mathematical expectations can be calculated, as advocated by F P Ramsey, Keynes's conventional coefficient, c, can be seen to present an inexact approximation of mathematical expectations once non linearity and non -additivity are integrated into a decision rule by the use of Keyes's weights [2w/(1+w)] and [1/(1+q)].

That Keynes's OAI line completely anticipates F P Ramsey's subjective approach to probability, while also showing how extremely narrow its application is. It also explains Kyburg's query as to why Ramsey had never mentioned this illustration at any time in his life when critiquing Keynes's logical theory of probability. Ramsey never mentioned Keynes's illustration because he knew exactly what Keynes was saying -precise, numerical probability was a special case of a much more general and applicable system-imprecise probability.

Section 3. The Birkhoff-von Neumann logical structure

Consider the Birkhoff-von Neumann comments on partial orders(ordering):

"Implication as partial ordering. It was suggested above that in any physical theory involving a phase-space, the experimental propositions concerning a system …correspond to a family of subsets of its phase-space Σ, in such a way that "x implies y" (x and y being any two experimental propositions) means that the subset of Σ corresponding to x is contained set-theoretically in the subset corresponding to y… The present section will be devoted to corroborating this hypothesis by identifying the algebraic-axiomatic properties of logical implication with those of set inclusion.

It is customary to admit as relations of "implication" only relations satisfying

Sl: x implies x.

S2: If x implies y and y implies z, then x implies z.

S3: If x implies y and y implies x, then x and y are logically equivalent … A system satisfying S1-S3, and in which the relation "x implies y" is written x C y, is usually[15] called a "partially ordered system, " and thus our first postulate concerning propositional calculi is that the physical qualities attributable to any physical system form a partially ordered system." (Birkhoff and von Neumann (BvN), 1936, p.828).

Consider BvN on lattice structure dealing with lub's, glb's, the Meet and the Join:

"In any calculus of propositions, it is natural to imagine that there is a weakest proposition implying, and a strongest proposition implied by a given pair of propositions. In fact, investigations of partially ordered systems from different angles all indicate that the first property which they are likely to possess, is the existence of greatest lower bounds and least upper bounds to subsets of their elements." (BvN, 1936, p.829).

BvN then state S5 and S6, that a lattice exists if to any pair of x and y elements there correspond a meet (glb) and a join(lub).

BvN point out that there are formal identities, L1-L4, which

"…are well-known formal properties of *and* and *or* in ordinary logic. This gives an algebraic reason for admitting as a postulate (if necessary) the statement that a given calculus of propositions is a lattice. "(BvN, 1936, p.829)

Continuing, BvN state:

"It is worth remarking that in classical mechanics, one can easily define the meet or join of any two experimental propositions as an experimental proposition-simply by having independent observers read off the measurements which either proposition involves, and combining the results logically. This is true in quantum mechanics only exceptionally-*only when all the measurements involved commute (are compatible)" (*BvN, 1936, p.829; italics added)

We now need to tie in the algebraic analogue of the "not" operator in propositional logic, which is called complementation:

"… Complemented lattices. Besides the (binary) operations of meet-and join-formation, there is a third (unary) operation which may be defined in partially ordered systems. This is the operation of complementation." (Bvn, 1936, p.830).

Thus, *complementation is the exact set-theory analogue of the logical operator "not" (negation).In propositional logic, negation is a unary operator applied to a statement or proposition*. It reverses the truth value of that proposition: if a statement P is true, then neg P is false, and vice versa. In set theory, complementation is a unary operation applied to a set A within a defined universal set U. The complement of A, denoted as A', is the set of all elements in U that are not in A.

The logical link between "not" and "complementation" arises from the correspondence between logic and set theory, where a proposition p is represented by a set (the set of all worlds where p is true). The logical connective "not", negation, corresponds to the set operation of complementation. The proposition, negative of p, corresponds to the set complement, not p. In this context, the concepts are fundamentally equivalent: classical negation in logic is algebraically a form of complementation in a Boolean algebra. They refer to the same underlying mathematical principle of inverting a value or selection but applied in different logical and mathematical formal systems.

BvN now turn *to the central difference between the classical standard approach and the non -standard approach required by quantum mechanics-the" … distributive identity of the propositional calculus*:

"…L6: a U (b ∩ c) = (a U b) ∩ (a U c) and a ∩ (b U c) = (a ∩ b) U (a ∩ c) *which is a law in classical, but not in quantum mechanics*." (BvN, 1936, p.830; italics added).

This is the point at which Birkhoff and von Neumann needed Keynes's interval valued probability, so as to deal with the uncertainty regarding outcomes depending on whether the application of the distributive law is possible.

Boolean-Keynesian interval-valued probabilities are non-distributive (non linear, non-additive or sub-additive).

Standard probability measures are defined by axioms that include additivity or countable additivity, meaning that the probability of a union of disjoint events is the sum of their individual probabilities.

Interval-valued probabilities, often associated with "modern" systems like Dempster-Shafer theory or Walley's imprecise probabilities, are a more general framework used when information is insufficient to assign a single, precise probability value. Instead, the probability is represented by a range (an interval) defined by a lower probability and an upper probability. (Note that, with the exception of von Neumann and Hailperin, the Boole and Keynes contributions are not mentioned)

This framework relaxes the strict additivity condition of classical probability theory. The measure associated with interval probabilities is often a non-additive measure, such as a Dempster -Shafer belief function or a possibility measure, as the Boolean -Keynesian results are simply still not known in 2026, which is 100 to 175 years after they were obtained by Boole and Keynes, respectively. In these systems, the probability assigned to the union of disjoint sets is typically less than or equal to the sum of the probabilities of the individual sets, which demonstrates a departure from the strict additivity or distributivity of classical probability.

Thus, Interval-valued probabilities themselves are ranges of values within the standard commutative framework of classical probability theory. As such, they are not inherently "non-commutative" in the mathematical sense. However, they can be used to quantify uncertainty in systems where the underlying observables are non-commuting, as happens in quantum mechanics.

BvN can thus use Interval-valued probabilities, when a scientist is unwilling to be more exact about the likelihood of an event than a certain interval. The arithmetic operations on these intervals (e.g., in a probability space context) are *typically treated using standard, commutative operations*.

Non-commutative probability is a sophisticated mathematical framework that arises in quantum mechanics and the study of random matrices. In this setting, the "random variables" are represented by non-commuting operators (like matrices where the order of multiplication matters: A X B does not equal B X A), and not by a single probability value or interval. The expectation is a linear functional on a non-commutative algebra, the result of which is a complex number, not an interval. The primary connection is that interval-valued probabilities can serve as a tool to describe joint distributions of non-commuting observables. Thus, while a precise (additive) joint probability distribution might not exist for non-commuting observables in quantum mechanics, the joint distribution can be quantified via upper and lower probabilities (an interval). This uses the concept of interval-valued probabilities to deal with the inherent non-commutativity of the quantum mechanical system.

In summary, the concept of non-commutativity (the order of operations matters) is a property of the underlying system or algebraic structure, not a property of the probability interval itself. The intervals are the result of applying a specific mathematical framework (imprecise probability) to describe such a system. It is known that non-commuting observables in quantum mechanics do not have joint probability.

In the Boole-Keynes system, the estimation, as opposed to exact calculation, of mathematical expectations, called rational expectations by Muth and Lucas, are a very special case, represented by the line OAI in the mathematical lattice illustration on p.39 of the TP, that holds only under the assertion that all probabilities are linear and additive, which is precisely which Keynes, rightly, rejected in the TP in chapter XXVI, pp.310-315 and in the GT on pp.136-137 and pp.161-163, See the first page of Keynes's 1937 *Eugenics Review* article for Keynes's emphatic rejection of the general applicability of both mathematical expectations, as asserted by F P Ramsey, and rational expectations, as asserted by Muth and Lucas.

Garrett Birkhoff, from 1933 to 1940, reformulated, revised and updated this topic, which was discovered and applied by Boole and Keynes. He gave a name to this field of mathematics. The name was Lattice Theory. Birkhoff also created new names for old topics when he started talking about posets, which he was led to do by his study of Hasse diagrams, the majority of which are lattice structures. Similarly, his introduction of the Meet (intersections and greatest lower bounds), of the Join (unions and least upper bound) and orthomodular lattices in Quantum Logic were named by Birkhoff. An orthomodular lattice structure is a generalization of the standard Boolean algebra, used to model the logical propositions of quantum mechanics. However, the standard Boolean approach was not able to deal with Boole's indeterminate probabilities because Jevons and Peirce eliminated Boole's partially defined operations and uninterpretable intermediate steps. The set of all closed subspaces of a Hilbert space forms an orthomodular lattice, some of which are standard (precise) and some of which are imprecise(non-standard). In so doing, Birkhoff had technically allowed for the future handling of non-complemented lattices. which is what Jevons and Peirce had eliminated. This would then allow mathematicians to handle Boole's indeterminate probability problems in a more "modern" way,

von Neumann stated in 1937 that he was using Keynes's logical (Keynes's non numerical, interval probabilities; Boole's indeterminate, interval valued probabilities) probabilities to model quantum probability:

"To see how von Neumann's thinking on the foundations of probability changed, we turn next to an unfinished manuscript from about 1937, which is included in his Collected Works [44] … Von Neumann then makes the following declaration:

"We prefer, therefore, to disclaim any intention to interpret the relations P (a, b) =θ (0<θ<1) in terms of strict logics. In other words, we admit:

Probability logics cannot be reduced to strict logics, but constitute an essentially wider system than the latter, and statements of the form P (a, b) =θ (0<θ<1) are perfectly new and sui generis aspects of physical reality.

So, probability logic appears as an essential extension of strict logics. This view, the so-called 'logical theory of probability', is the foundation of J. N. [sic] Keynes's work on the subject."

In short, the later von Neumann interprets quantum probabilities as logical probabilities. Moreover, he explicitly identifies this view with that worked out by Keynes. "(Stacey, 2016, pp.3-4, italics added).

Of course, logical probabilities are interval valued (indeterminate) probabilities and the logical theory of probability uses interval valued probability /decision weights in order to create a 'new logic of uncertainty'. (Hishiyama, 1969). By analogy, Birkhoff and von Neumann in 1936 are following Keynes (who was following Boole) in terms of creating the logical foundations for analyzing and incorporating Heisenberg's uncertainty principle into Physics, which requires ruling out the applicability of the distributive law, which then leads to non-additivity and non-linearity in probability assessments. While Birkhoff and von Neumann generally succeeded, Keynes did not succeed in the field of economics, which is still mired in the same type of analysis found in physics between the mid 1730's and the mid 1870's, which required the standard probability logic of strict complementation needed to implement the Benthamite Utilitarian framework originally outlined by Jeremy Bentham in 1787.

Section 4. A reassessment of economist views of Keynes's mathematical skill set in light of von Neumann's assessments

The Post Keynesian view of Keynes's mathematical skills was that he was a very poor mathematician whose work was based on Platonic intuition. The latest such assessment comes from Basili (2024) and Basili & Pratelli (2024):

Consider the following claims made by Basili in (Basili, 2024):

"The crucial question is about the possibility of including an interval probability measure in Keynes's theoretical framework.[15] Keynes published TP in 1921, Ramsey wrote the papers included in the Foundation of Mathematics between 1923 and 1929. In that period there was neither the axiomatic theory of lattice nor the mathematical representation of probability as a positive normed measure, which came later. The controversy between Keynes and Ramsey involves the foundation of algebra of logic and measure theory but, unfortunately, formal theory is incomplete, sparing or absent, at that time." (Basili, 2024, p.7) and "In 1933, Birkhoff introduces the term lattice to describe an independent mathematical theory. In 1936 in two papers about an alternative space model for quantum mechanics and new geometrical structures, von Neumann gave a definition of lattice. In 1936, in the context of the Einstein-Pocholsky-Rosen and Bohr's controversy about the quantum mechanical description of physical reality, Birkhoff and von Neumann published The Logic of Quantum Mechanics and developed a lattice theory that was not distributive but ortho-complemented, only.[23] Finally, Birkhoff shows some lattice applications (1938) and publishes the first monography on lattice theory, in 1940. Summarizing, lack of a formal definition of a consistent interval probability measure is a problem related to Keynes's knowledge of mathematics or is the limit of contemporary mathematical theory? Keynes was not able to be precise and formal in his definition of probability under uncertainty represented by closed intervals of probabilities, because such a mathematics had not been discovered, simply." (Basili, 2024, p10).

Basili's (also see Basili and Pratelli (2024), Basili and Zappia (2021) and Basili and Zappia (2009)) view is representative of the Post Keynesian claims associated with the Cambridge Journal of Economics. Currently, the leading Post Keynesian is Lord Robert Skidelsky.

Skidelsky's dubious claims about Keynes's work in his TP, which were contained in Skidelsky's 1992 second volume on Keynes, occur throughout chapter 3 on pages 56-89. Skidelsky demonstrates a lack of understanding about the mathematical, logical, statistical and probabilistic foundations underlying Keynes's presentation of his logical theory of probability in his A Treatise on Probability. Supposedly, according to Skidelsky, F P Ramsey, in both of his reviews of 1922 and 1926 of Keynes's TP, was able to prove that there were many major, logical errors in Keynes's work. Skidelsky made the following overall assessment:

"After an hour or so of beautiful demolition work little of the baroque edifice of the Treatise was left standing ". (Skidelsky, 1992, p.70; note that the demolition work is being done by Ramsey according to Skidelsky).

In fact, pace Skidelsky and his Post Keynesian supporters (see, for example, Bateman, Backhouse, Clarke, Coates, Davis, Runde, O'Donnell, Gillies, Mellor, Methven, Misak), nothing written by Ramsey on Keynes's A Treatise on Probability between 1921 and 1926 is correct. Ramsey's basic, major error, which he repeated many times between 1922 and 1926, was to assume that Keynes had based his A Treatise on Probability on Plato and G.E Moore's Principia Ethica, a book devoted to the study of ethics, morals, religion, aesthetics, art, and metaphysics, not probability and statistics. In fact, everything in the A Treatise on Probability was based on only one major source - Boole's 1854 The Laws of Thought.

Note that the assessments of Basili and Basili and Pratelli and the Post Keynesians directly conflict with the assessment of von Neumann.

Section 5. Conclusions

A careful reader of the BvN Oct., 1936 paper, who has also carefully read Keynes's A Treatise on Probability, but not Boole's The Laws of Thought, will be led inexorably to the conclusion that von Neumann's 1937 comments about Keynes were made in regard to the format used in the Oct., 1936 paper, which he wrote with Birkhoff. Von Neumann's mastery of Keynes's TP would have been expected by an intellectual giant of von Neumann's caliber.

This can be compared to an assessment made by I. Hishiyama, that Keynes's TP had never been read (Hishiyama, 1969). Since Hishiyama's conclusion was reached in 1969, the so called "Keynes scholars" have attempted to read and understand Parts I and III of the TP. Parts II and V are simply skipped. Unfortunately, unless you are talking about an F Y Edgeworth or H. Townshend, the assessments of economists, and also philosophers and historians, are erroneous, as they attempt to combine Parts I and III of the TP with the erroneous claims made about Keynes's TP by F P Ramsey ("…Keynes's mysterious logical probabilities…"), I J Good or Richard Braithwaite. The result is an ever growing plethora of conflicting "interpretations" about "What Keynes Really Must Have Meant". The "Keynes scholars" unanimously agree only that Keynes's TP theory is, at best, an ordinal theory of comparative probability, that is operational only some of the time, Keynes's theory of evidential weight is "mysterious" (Brekel, 2025) and "a very confused problem" (Brekel, 2022).

The only conclusion possible is that the work of the so called "Keynes scholars" is dubious. It consists of many thousands of articles and books, starting with the publication of R B Braithwaite's 1973 editorial foreword to the 1973 CWJMK version of the TP and E Roy Weintruab's 1975 HOPE article, that completely misinterpret Keynes's lattice diagram on p.39 of the TP. It is interesting that Ramsey's numerous characterizations of Keynes's work as being "mysterious" matches Jevons's characterizations of Boole's work as being "mysterious".

In closing, I provide another assessment of von Neumann's, which makes it clear that von Neumann does not find Keynes's approach to be "mysterious "at all:

"In 1938, von Neumann (Stacy, 2016, p.5) gives the following restated assessment of von Neumann's unpublished, 1937 statement:

"A complete derivation of quantum mechanics is only possible if the propositional calculus of logics is so extended, as to include probabilities, in harmony with the ideas of J. M. Keynes. In the quantum mechanical terminology: the notion of a « transition probability »from a to b, to be denoted by P(a,b) must be introduced. P(a, b) is the probability of b, if a is known to be true. P(a,b) can be used to define a ≦b and −a : P(a,b) =1 means a ≦b, P(a,b) =0 means a≦−b. (But P(a,b) =φ, with a φ>0, <1 is a new « sui generis » statement, only understandable in terms of probabilities.) [46, p.38; italics added]." (Stacey, 2016, p.6)

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